

Thick Domain Walls and Charged Dilaton Black Holes

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We study a black hole domain wall system in dilaton gravity which is the low-energy limit of the superstring theory. We solve numerically equations of motion for real self-interacting scalar field and justify the existence of static axisymmetric field configuration representing the thick domain wall in the background of a charged dilaton black hole. It was also confirmed that the extreme dilaton black hole always expelled the domain wall.

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I. INTRODUCTION

It is believed that the early universe undergoes a series of vacuum phase transitions which led to several types of topological defects [1]. Topological defects arising in the early universe could envisage the high-energy phenomena that were beyond the range of our accelerators. The studies of topologically nontrivial field configurations in the background of a black hole attract great interest in recent years. In Refs.[2] it was shown both analytically and numerically that an Abelian-Higgs vortex could act as a long hair for the Schwarzschild and Reissner-Nordström (RN) black hole solution.

The problem of dilaton black hole cosmic string system was considered in Refs.[3, 4]. It was revealed that the horizon of a charged dilaton black hole could support a long-range fields of the Nielsen-Olesen type, which one could consider as black hole hair. If the dilaton black hole approached to extremality one could show that the vortex was always expelled from it.

De Villier *et al.* [5] studied the dynamics of the scattering and capturing process of an infinitely thin cosmic string in the background of Schwarzschild black hole spacetime.

The domain wall in the black hole background was considered by Christensen *et al.* [6], showing that there exists family of infinitely thin walls intersecting black hole event horizon.

In Ref.[7] the stability of a Nambu-Goto membrane at the equatorial plane in the background of RN-de Sitter spacetime was studied. It was shown that a membrane intersecting charged black hole is unstable and the positive cosmological constant strengthens this instability.

The gravitational interaction of a thick domain wall in the Schwarzschild black hole background was studied in Ref.[8]. Bonjour *et al.* [9] have investigated the spacetime of thick gravitating domain wall with local planar symmetry and reflection symmetry around the wall's core. They revealed that the domain wall spacetime has a cosmological horizon as in the de Sitter case.

Recently, the interaction of black holes and extended objects attracts the attention in the context of superstring/M-theory and the brane world scenario [10]. It is argued that in this scenario black holes on the gravitating membrane are realized as *black cigars* in the bulk spacetime intersecting membrane. In Ref.[11] the problem of black hole on a topological domain wall (including the gravitational back-reaction) was considered. In [12] the behaviour of the domain wall in the spacetime of a dilaton black hole was analyzed and it was shown analytically that the extreme dilaton black hole always expelled the domain wall.

In our paper we shall provide some continuity with the previous work [12] and we shall consider the interaction between a domain wall and a charged dilaton black hole taking into account the thickness of the domain wall and the potential of the scalar field of φ^4 and sine-Gordon forms.

The outline of the paper is as follows. Sec.II is devoted to the general analytic considerations of the domain wall in the spacetime of the charged dilaton black hole. In Sec.III we presented the numerical analysis of the equation of motion for two cases of potentials with discrete set of degenerate minima, i.e., the φ^4 and sine-Gordon potentials. In Sec.IV we finish with the general summarizing of our work.

II. THE BASIC EQUATIONS OF THE PROBLEM

In this section we shall consider a static thick domain wall in the background of a charged dilaton black hole. This black hole is a static spherically symmetric solution of Einstein-dilaton gravity being the low-energy limit of the superstring theory. In our considerations we assume that the domain wall is constructed by means of a self-interacting scalar field in the considered background. The metric of a charged dilaton black hole may be written as [13]

$$ds^2 = - \left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r}\right)} + r \left(r - \frac{Q^2}{M}\right) (d\theta^2 + \sin^2 \theta d\varphi^2), \quad (1)$$

where we define $r_+ = 2M$ and $r_- = \frac{Q^2}{M}$ which are related to the mass M and charge Q by the relation $Q^2 = \frac{r_+ r_-}{2} e^{2\phi_0}$. The charge of the dilaton black hole Q , couples to the field $F_{\alpha\beta}$. The dilaton field is given by $e^{2\phi} = \left(1 - \frac{r_-}{r}\right) e^{-2\phi_0}$, where ϕ_0 is the dilaton's value at $r \rightarrow \infty$. The event horizon is located at $r = r_+$. For $r = r_-$ is another singularity, one can however ignore it because $r_- < r_+$. The extremal black hole occurs when $r_- = r_+$, when $Q^2 = 2M^2 e^{2\phi_0}$.

We consider a general matter Lagrangian with real Higgs field and the symmetry breaking potential of the form as follows:

$$\mathcal{L}_{dw} = -\frac{1}{2} \nabla_\mu \varphi \nabla^\mu \varphi - U(\varphi). \quad (2)$$

The symmetry breaking potential $U(\varphi)$ has a discrete set of degenerate minima. The energy-momentum tensor for the domain wall yields

$$T_{ij}(\varphi) = -\frac{1}{2} g_{ij} \nabla_m \varphi \nabla^m \varphi - U(\varphi) g_{ij} + \nabla_i \varphi \nabla_j \varphi. \quad (3)$$

For the convenience we scale out parameters via transformation $X = \varphi/\eta$ and $\epsilon = 8\pi G\eta^2$. The parameter ϵ represents the gravitational strength and is connected with the gravitational interaction of the Higgs field. Defining $V(X) = \frac{U(\varphi)}{V_F}$, where $V_F = \lambda\eta^4$ we arrive at the following expression:

$$8\pi G \mathcal{L}_{dw} = -\frac{\epsilon}{w^2} \left[w^2 \frac{\nabla_\mu X \nabla^\mu X}{2} + V(X) \right], \quad (4)$$

where $w = \sqrt{\frac{\epsilon}{8\pi G V_F}}$ represents the inverse mass of the scalar after symmetry breaking, which also characterize the width of the wall defect within the theory under consideration. Having in mind (4) the equations for X field may be written as follows:

$$\nabla_\mu \nabla^\mu X - \frac{1}{w^2} \frac{\partial V}{\partial X} = 0. \quad (5)$$

In the background of the dilaton black hole spacetime the equation of motion for the scalar field X implies

$$\frac{1}{r \left(r - \frac{Q^2}{M} \right)} \partial_r \left[\left(r - \frac{Q^2}{M} \right) (r - 2M) \partial_r X \right] + \frac{1}{r \left(r - \frac{Q^2}{M} \right) \sin \theta} \partial_\theta \left[\sin \theta \partial_\theta X \right] = \frac{1}{w^2} \frac{\partial V}{\partial X}. \quad (6)$$

Having in mind relation (3) one can define the energy density of scalar fields φ in the form

$$E = \frac{T_t^t}{\lambda \eta^4} = \left[-\frac{1}{2} (X_{,r})^2 \left(1 - \frac{2M}{r} \right) - \frac{1}{2} (X_{,\theta})^2 \frac{1}{r \left(r - \frac{Q^2}{M} \right)} \right] w^2 - V(X). \quad (7)$$

In our considerations we take into account for two cases of potentials with a discrete set of degenerate minima, namely – the φ^4 potential

$$U_1(\varphi) = \frac{\lambda}{4} (\varphi^2 - \eta^2)^2, \quad (8)$$

– the sine-Gordon potential

$$U_2(\varphi) = \lambda \eta^4 [1 + \cos(\varphi/\eta)]. \quad (9)$$

III. BOUNDARY CONDITIONS AND NUMERICAL INTEGRATION.

A. Boundary conditions for φ^4 potential

Because of the fact that the charged dilaton black hole is asymptotically flat, the asymptotic boundary solution of equation of motion for potential (8) is the solution of equation of motion in flat spacetime, namely

$$\varphi_1(z) = \eta \tanh(\sqrt{\lambda/2} \eta z). \quad (10)$$

In our units this gives

$$X(r, \theta) = \tanh \left(\frac{r \cos \theta}{\sqrt{2} w} \right) \quad (11)$$

In this case $V(X) = \frac{1}{4}(X^2 - 1)^2$ and $\partial V/\partial X = X(X^2 - 1)$ so the equation of motion (6) takes the form

$$\frac{1}{r \left(r - \frac{Q^2}{M} \right)} \partial_r \left[\left(r - \frac{Q^2}{M} \right) (r - 2M) \partial_r X \right] + \frac{1}{r \left(r - \frac{Q^2}{M} \right) \sin \theta} \partial_\theta \left[\sin \theta \partial_\theta X \right] - \frac{1}{w^2} X(X^2 - 1) = 0. \quad (12)$$

while the energy density is equal to the following expression:

$$E = \left[-\frac{1}{2} (\partial_r X)^2 \left(1 - \frac{2M}{r} \right) - \frac{1}{2} (\partial_\theta X)^2 \frac{1}{r \left(r - \frac{Q^2}{M} \right)} \right] w^2 - \frac{1}{4} (X^2 - 1)^2. \quad (13)$$

On the horizon relation (12) gives the boundary condition

$$\frac{1}{2M} \partial_r X|_{r=2M} = \frac{-1}{2M \left(2M - \frac{Q^2}{M} \right) \sin \theta} \partial_\theta [\sin \theta \partial_\theta X] + \frac{1}{w^2} X(X^2 - 1). \quad (14)$$

Because we consider here only the case when the core of the wall is located in the equatorial plane $\theta = \pi/2$ of the black hole we impose the Dirichlet boundary condition at the equatorial plane

$$X|_{\theta=\pi/2} = 0. \quad (15)$$

The regularity of the scalar field on the symmetric axis requires the Neumann boundary condition on the z-axis:

$$\left. \frac{\partial X}{\partial \theta} \right|_{\theta=0} = 0. \quad (16)$$

Far from the black hole we want to obtain flat spacetime solution (10). Because our computational grid is finite this requires the following boundary condition to be imposed on the outer boundary of the grid:

$$X|_{r=r_{max}} = \tanh\left(\frac{r_{max} \cos \theta}{\sqrt{2}w}\right). \quad (17)$$

B. Boundary conditions for the sine-Gordon potential.

For the sine-Gordon potential (9) the flat spacetime solution is given by

$$\varphi_2(z) = \eta \left\{ 4 \arctan \left[\exp(\sqrt{\lambda} \eta z) \right] - \pi \right\}. \quad (18)$$

In our units this is equivalent to following:

$$X(r, \theta) = 4 \arctan \left[\exp \left(\frac{r \cos \theta}{w} \right) \right] - \pi. \quad (19)$$

In this case $V(X) = 1 + \cos(X)$ and $\partial V / \partial X = -\sin(X)$, and thus the equation of motion (6) takes the form

$$\frac{1}{r \left(r - \frac{Q^2}{M} \right)} \partial_r \left[\left(r - \frac{Q^2}{M} \right) (r - 2M) \partial_r X \right] + \frac{1}{r \left(r - \frac{Q^2}{M} \right) \sin \theta} \partial_\theta \left[\sin \theta \partial_\theta X \right] + \frac{1}{w^2} \sin(X) = 0, \quad (20)$$

while the energy density is given by the relation

$$E = \left[-\frac{1}{2} (\partial_r X)^2 \left(1 - \frac{2M}{r} \right) - \frac{1}{2} (\partial_\theta X)^2 \frac{1}{r \left(r - \frac{Q^2}{M} \right)} \right] w^2 - 1 - \cos(X). \quad (21)$$

On the horizon from Eq.(20) one has the boundary condition as follows:

$$\frac{1}{2M} \partial_r X|_{r=2M} = \frac{-1}{2M \left(2M - \frac{Q^2}{M} \right) \sin \theta} \partial_\theta [\sin \theta \partial_\theta X] - \frac{1}{w^2} \sin(X). \quad (22)$$

Of course, this must be accompanied by the Dirichlet boundary conditions at the equatorial plane of the black hole (15), and the Neumann boundary condition on the symmetry axis (16), and at the outer edge of the grid

$$X|_{r=r_{max}} = 4 \arctan \left[\exp \left(\frac{r_{max} \cos \theta}{w} \right) \right] - \pi. \quad (23)$$

C. Numerical integration.

In order to solve numerically Eq.(6) we used the numerical method previously used in Refs.[3]. Namely, we use overrelaxation method slightly modified to handle boundary conditions on the black hole horizon. We solve the

equation of motion on uniformly spaced polar grid (r_i, θ_i) with boundaries at $r_{min} = 2M$, outer radius $r_{max} \gg 2M$ (usually we use $r_{max} = 20M$), and θ ranging from 0 to $\pi/2$. The rest of the solution is obtained from symmetry of the scalar field – $X(r, -\theta) = X(r, \theta)$ and $X(r, \theta > \pi/2) = -X(r, \pi - \theta)$.

Figure 1 presents the results of numerical integration of equation of motion (12). On the same plot we show also the energy (13) for this scalar field configuration. The mass of the black hole is taken to be $M = 1$ and the charge $Q = 0.1$, the domain wall thickness $w = 1$. Fig.2 depicts the field X and the energy E for the extreme dilaton black hole with parameters $M = 1$, $Q = \sqrt{2}$ and the domain width $w = 1$. For this case one has the expulsion of the domain wall from the extremal dilaton black hole. The so-called *Meissner effect* was analytically predicted in Ref.[12]. Figs.3 - 4 were plotted for the same black hole parameters but we changed the domain wall width and put it $w = 10$. Then for this width of the domain wall the dilaton black hole is enveloped in the core region of the wall. Figs.5 - 6 show the values of X field and the energy for the dilaton and extremal dilaton black holes and the domain width $w = 1$. For this kind of potential one observes also the *Meissner effect* for the extreme black hole. In Figs.7 - 8 we take into account the domain width $w = 10$. In this case we have also enveloping of the black hole in the core region of the considered domain wall.

IV. CONCLUSIONS

In our paper we studied the problem of the domain wall in the vicinity of a charged dilaton black hole, being the static spherically symmetric solution of Einstein-dilaton gravity. We solve numerically Eqs. of motion for real scalar field with φ^4 and sine-Gordon potentials. We use the modified overrelation method modified to comprise the boundary conditions on the black hole event horizon. In our considerations we also use the parameter $w = 1/\sqrt{\lambda}\eta$, which characterizes the thickness of the domain wall.

We have justified the existence of static axisymmetric field configuration representing thick domain wall in the nearby of dilaton black hole for both φ^4 and sine-Gordon potential cases. We studied the specific black hole domain wall configuration, when the core of the domain wall is located at the equatorial plane of the black hole. As in Ref. [8] we assumed that the gravitational effect of the domain wall is negligible compared to the effect caused by a charged dilaton black hole. In the case of the extreme dilaton black hole we find that the domain wall is always expelled from the considered black hole, justifying the analytical predictions presented in Ref. [12]. This behaviour envisages the analog of the so-called *Meissner effect* for the extreme dilaton black hole. This effect was also revealed in the case of another topological defect, i.e., cosmic string which was also expelled from the extremal dilaton black hole [3, 4].

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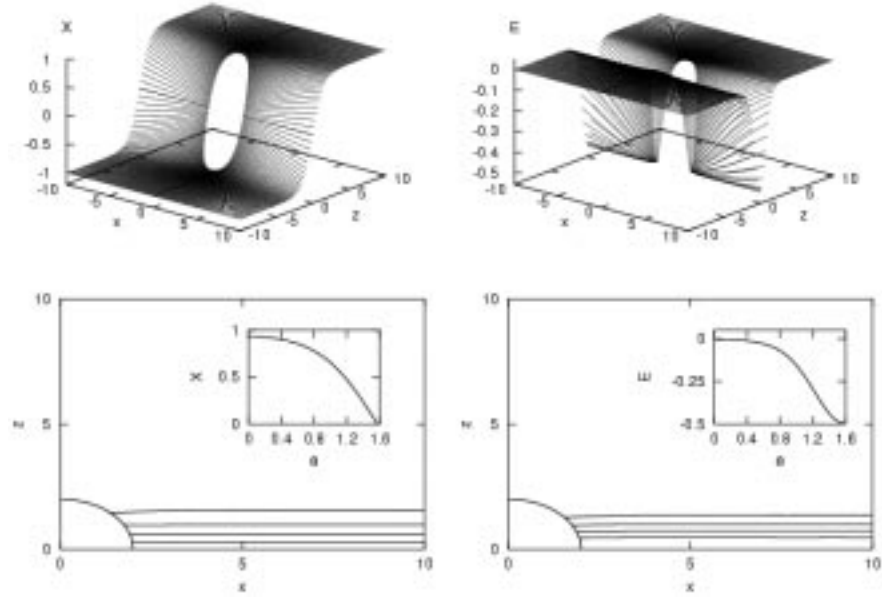


FIG. 1: The field X (left panels) and the energy E (right panels) for the ϕ^4 potential. Isolines on bottom panels are drawn for 0.2, 0.4, 0.6 and 0.8 for the field X and for -0.1 , -0.2 , -0.3 and -0.4 for the energy. Inlets in bottom plots show the value of the fields on the black hole horizon. Black hole has $M = 1.0$, $Q = 0.1$ and the domain width is $w = 1.0$.

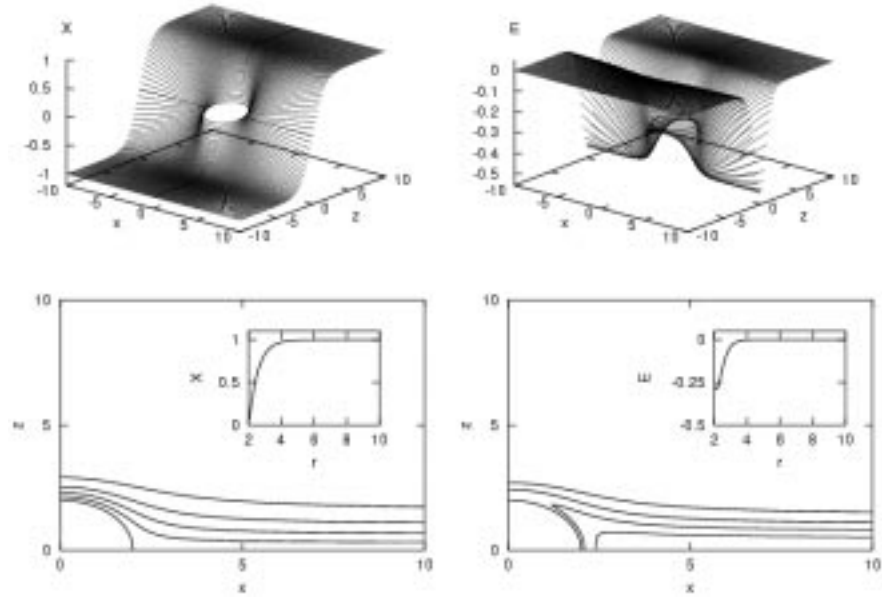


FIG. 2: The field X (left panels) and the energy E (right panels) for the ϕ^4 potential and extreme black hole. Isolines on bottom panels are drawn for 0.2, 0.4, 0.6 and 0.8 for the field X and for -0.1 , -0.2 , -0.3 and -0.4 for the energy. Inlets in bottom plots show the value of the fields on the black hole z -axis. Black hole has $M = 1.0$, $Q = \sqrt{2}$ and the domain width is $w = 1.0$.

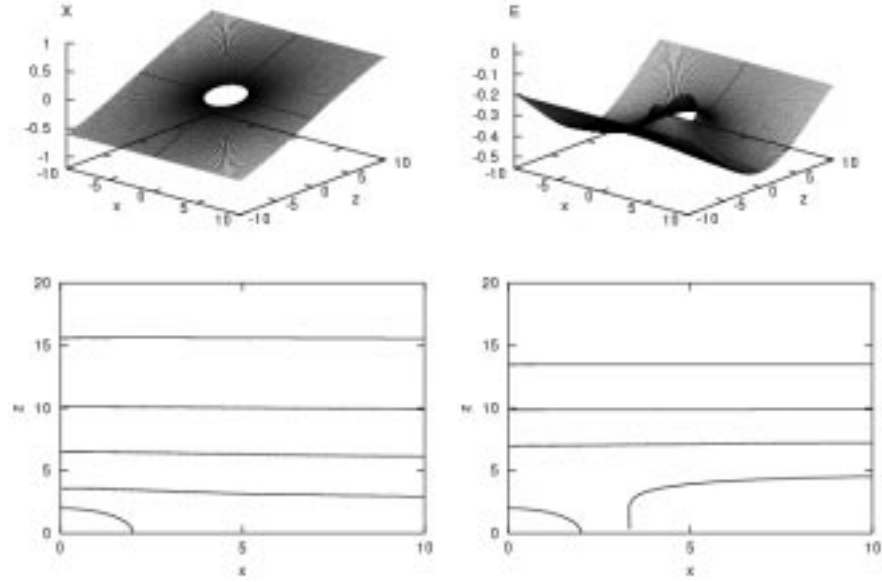


FIG. 3: The field X (left panels) and the energy E (right panels) for the ϕ^4 potential. Isolines on bottom panels are drawn for 0.2, 0.4, 0.6 and 0.8 for the field X and for -0.1, -0.2, -0.3 and -0.4 for the energy. Black hole has $M = 1.0$, $Q = 0.1$ and the domain width is $w = 10.0$.

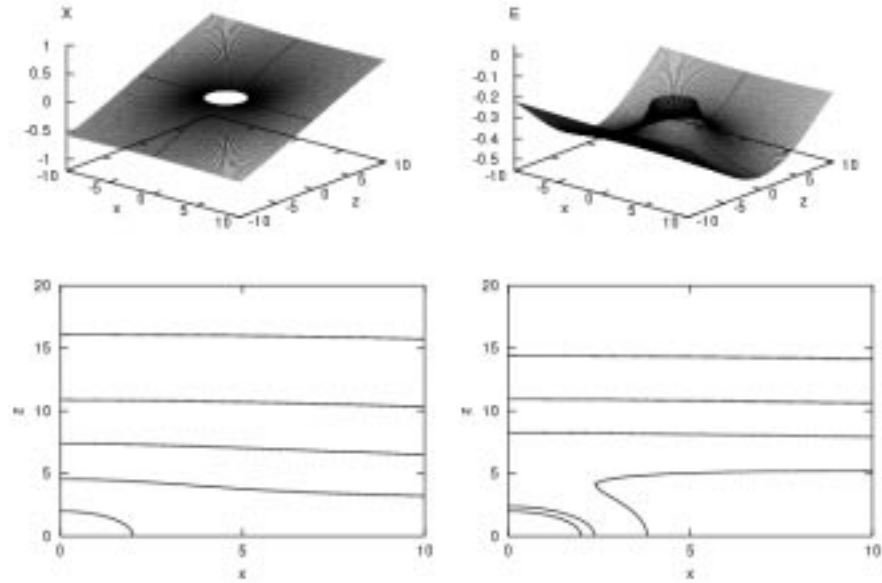


FIG. 4: The field X (left panels) and the energy E (right panels) for the ϕ^4 potential and extreme black hole. Isolines on bottom panels are drawn for 0.2, 0.4, 0.6 and 0.8 for the field X and for -0.1, -0.2, -0.3 and -0.4 for the energy. The energy isoline around the black hole is -0.3. Black hole has $M = 1.0$, $Q = \sqrt{2}$ and the domain width is $w = 10.0$.

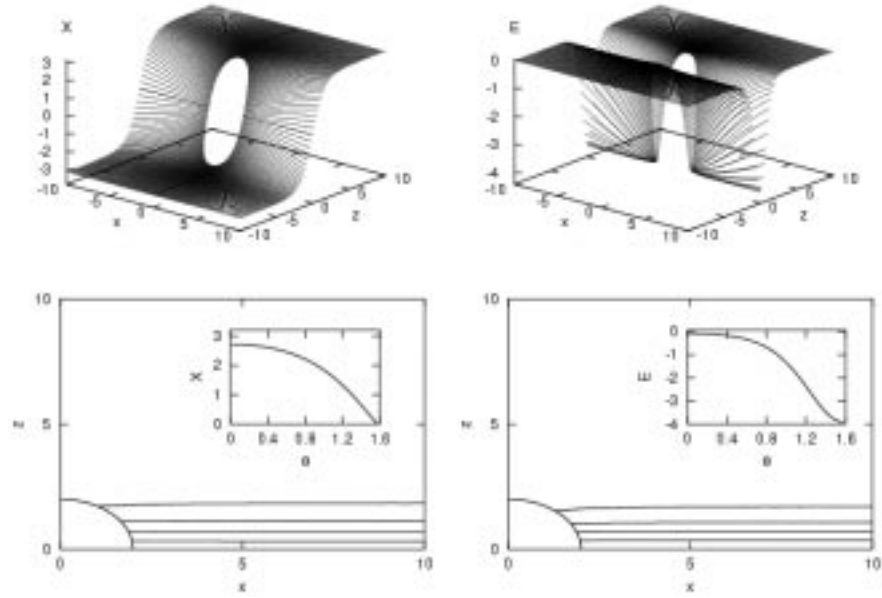


FIG. 5: The field X (left panels) and the energy E (right panels) for the sine-Gordon potential. Isolines on bottom panels are drawn for $0.2\pi, 0.4\pi, 0.6\pi$ and 0.8π for the field X and for $-0.5, -1.5, -2.5$ and -3.5 for the energy. Inlets in bottom plots show the value of the fields on the black hole horizon. Black hole has $M = 1.0$, $Q = 0.1$ and the domain width is $w = 1.0$.

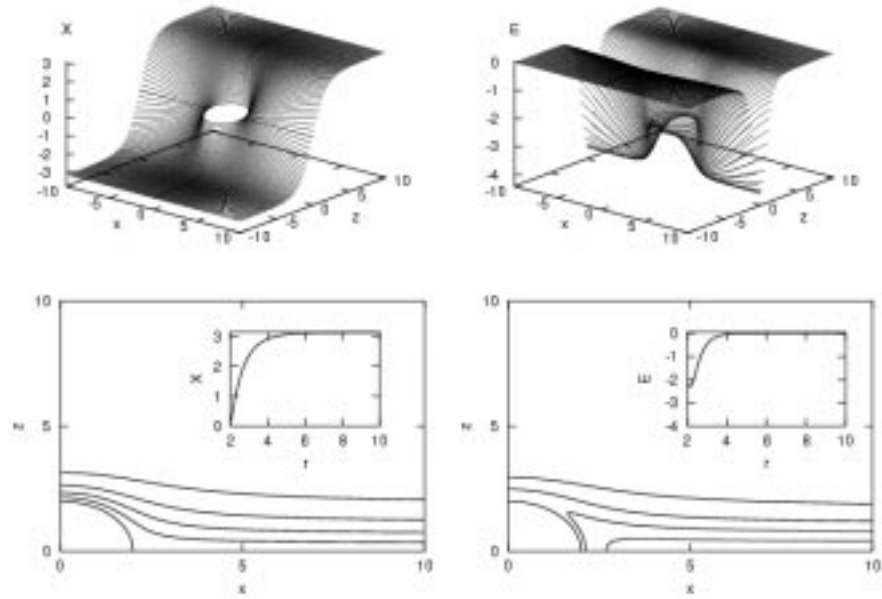


FIG. 6: The field X (left panels) and the energy E (right panels) for the sine-Gordon potential. Isolines on bottom panels are drawn for $0.2\pi, 0.4\pi, 0.6\pi$ and 0.8π for the field X and for $-0.5, -1.5, -2.5$ and -3.5 for the energy. Inlets in bottom plots show the value of the fields on the black hole z -axis. Black hole has $M = 1.0$, $Q = \sqrt{2}$ and the domain width is $w = 1.0$.

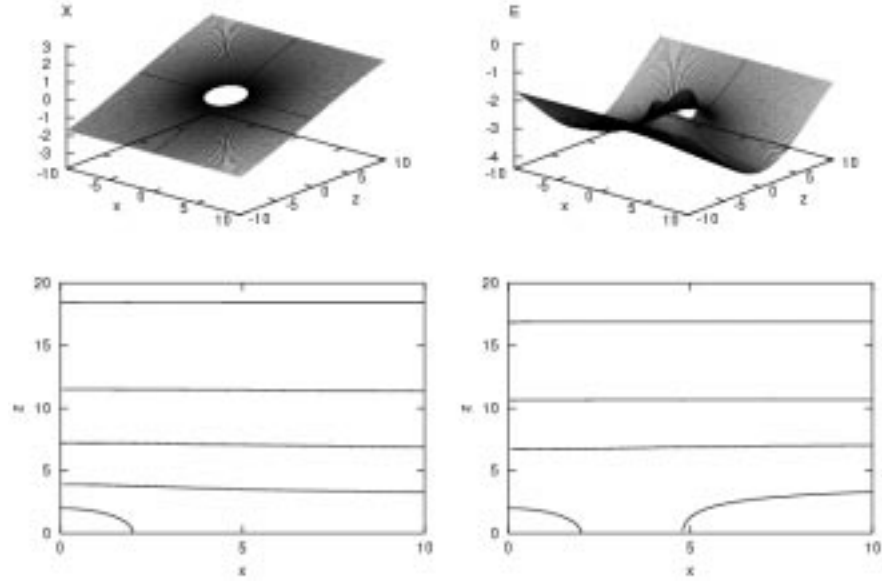


FIG. 7: The field X (left panels) and the energy E (right panels) for the sine-Gordon potential. Isolines on bottom panels are drawn for $0.2\pi, 0.4\pi, 0.6\pi$ and 0.8π for the field X and for $-0.5, -1.5, -2.5$ and -3.5 for the energy. Black hole has $M = 1.0$, $Q = 0.1$ and the domain width is $w = 10.0$.

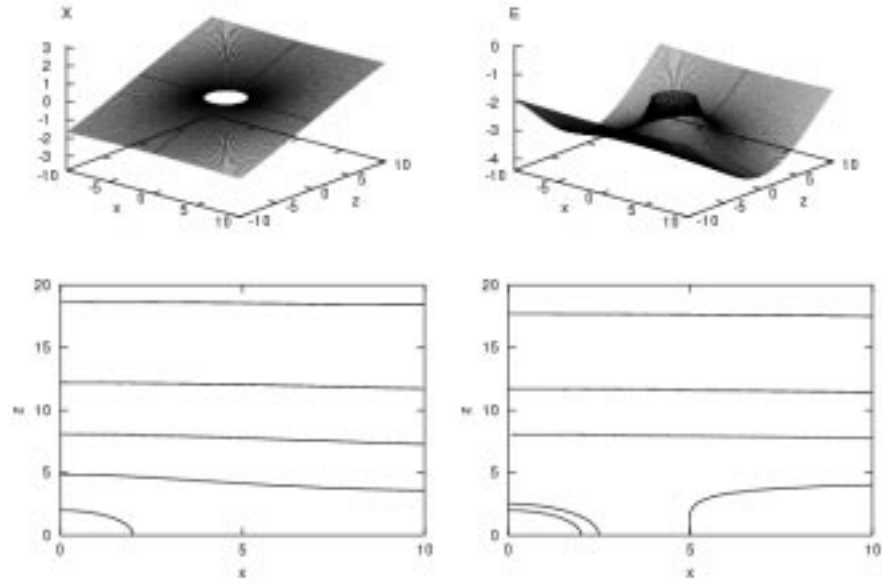


FIG. 8: The field X (left panels) and the energy E (right panels) for the sine-Gordon potential. Isolines on bottom panels are drawn for $0.2\pi, 0.4\pi, 0.6\pi$ and 0.8π for the field X and for $-0.5, -1.5, -2.5$ and -3.5 for the energy. The energy isoline around the black hole is -2.5 . Black hole has $M = 1.0$, $Q = \sqrt{2}$ and the domain width is $w = 10.0$.